Template Based Control of Hexapedal Running

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Abstract

In this paper, we introduce a hexapedal locomotion controller that simulation evidence suggests will be capable of driving our RHex robot at speeds exceeding five body lengths per second with reliable stability and rapid maneuverability. We use a low dimensional passively compliant biped as a "template" — a control target for the alternating tripod gait of the physical machine. We impose upon the physical machine an approximate inverse dynamics within-stride controller designed to force the true high dimensional system dynamics down onto the lower dimensional subspace corresponding to the template. Numerical simulations suggest the presence of asymptotically stable running gaits with large basins of attraction. Moreover, this controller improves substantially the maneuverability and dynamic range of RHex's running behaviors relative to the initial prototype open-loop algorithms.

1 Introduction

This paper concerns a new hexapedal running controller that promises to improve on the performance of prototype open-loop algorithms that presently drive our experimental hexapod robot, RHex [18]. Our emphasis on running is primarily motivated by the speed and efficiency afforded by dynamical modes of operation which are very difficult to achieve with traditional, statically stable gaits for hexapedal robots [8, 11, 21]. Raibert's runners [15, 16] first demonstrated the advantages of such dynamical gaits in surpassing the performance of purely kinematic algorithms. Later examples include the Scout class of quadrupeds [4, 5] and brachiating robots [14].

Over the last three decades, research in biomechanics [1, 7] has revealed that simple spring-mass models describe accurately the running motions of animals with different sizes and morphologies [2, 3, 9]. Recently, a more formal model, the Spring-Loaded Inverted Pendulum(SLIP) has been introduced as a useful tool in characterizing basic aspects of running, including stability and parameterizations of stable gaits [12, 20]. In this paper, we proceed one step further and adopt the SLIP model as a literal control target for running.

Toward this end, we introduce a bipedal extension to the basic SLIP model as a "template" — a simple dynamical system capturing the characteristic features of the task at hand [10]. In particular, the presence of two legs represents the alternating tripod gait which we adopt for our running controller. The lack of radial leg actuation in our experimental hexapod imposes a focus on explicit leg recirculation (rather than protraction) strategies leading to the introduction of a novel mechanism for its coordination. A natural correspondence between the passive radial compliance in RHex's legs and the SLIP model greatly reduces the active control effort required to achieve the target dynamics of the template.

Our resulting control architecture is an elaborated version of the template/anchor hierarchy of [19] with two levels. On "top" is a stride-to-stride level deadbeat controller affording a relatively simple task level interface for the command of mass center speed and height. Commands to the SLIP template impose carefully chosen parametric variations in a within-stride continuous time approximate inverse dynamics based hip torque controller that lies "beneath", attempting to force the dynamics of the robot to mimic the template as closely as possible. In the following sections, we provide systematic numerical evidence to suggest that the combination of these controllers will be capable of achieving reliably stable but highly maneuverable hexapedal running over a large range of speeds.

2 The Bipedal SLIP template

RHex's morphology introduces a number of fundamental constraints on feasible locomotion controllers. Most importantly, the limitation to one actuator per leg for a full 24 degree of freedom mechanism (see [18]) imposes a severe degree of "underactuation", significantly exacerbated by the kinematic singularities around the standard operating configuration. As a re-

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sult, controllers must rely on dynamic properties of the system, particularly the radial compliance in the legs, to achieve reasonable performance and range of behaviors. Our choice of template needs to capture these properties and limitations to ensure that its dynamics can be achieved with RHex's morphology.

In designing our controller, we primarily concentrate on the alternating tripod gait, which is adopted by the majority of hexapedal insects at high speeds [22]. In this gait, two tripods operate out of phase with each other, but are internally synchronized. The resulting pattern describes a "virtual bipedal" gait, motivating a planar compliant biped as the template for our locomotion controllers. In this section, we briefly review this template and its associated controllers. A much more complete treatment can be found in [17].

2.1 Hybrid System Model

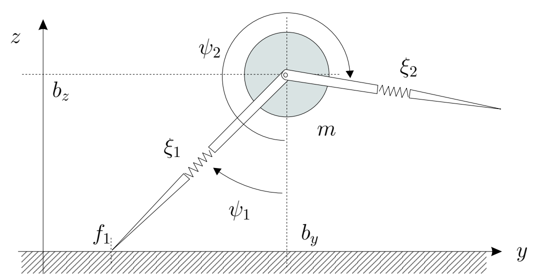

Figure 1: The Bipedal Spring-Loaded Inverted Pendulum(BSLIP) Model.

Figure 1 illustrates the planar Bipedal Spring-Loaded Inverted Pendulum (BSLIP) model. It consists of a point mass m, attached to two compliant massless legs that can freely rotate around the hip joint. Both legs incorporate passive springs as well as viscous damping. The mass is constrained to remain in the saggital plane, and is acted upon by gravity. Each leg has two alternating discrete modes — *stance* and *swing*.

Throughout the stance phase of a leg, its toe is fixed on the ground. When the legs are in their swing phase, however, they do not affect the body dynamics. Their length and angle is governed by fully actuated first order dynamics, through which the touchdown angle and precompression can be controlled.

2.2 Control of Bipedal Gaits

Our bipedal locomotion controller has three major components: a finite state machine(FSM) to enforce leg alternation, a gait controller to regulate speed and height through proper choice of touchdown leg states as well as a recirculation controller to synchronize the stance and swing legs. For space considerations, this section only gives a brief overview of these components. Further details can be found in [17].

As a complement to the physical *modes* of a leg, discussed previously, the leg recirculation controller undertakes a succession of three states: *active*, *idle* or *recirculate*, givrened by a separate FSM. The leg is active when it is in contact with the ground. It becomes idle when it lifts off and remains so until the touchdown of the other leg. Finally, the leg recirculates in order to achieve the desired touchdown states.

In the spirit of Raibert's runners, our controller regulates the speed and height of locomotion through a discrete set of command inputs for each step: touchdown angle, touchdown length (precompression) and liftoff length. This choice of parameters is compatible with the radially passive nature of RHex's legs. Similar to our earlier work with a different set of inputs [19], we use a deadbeat strategy based on approximate plant inversion as our gait controller.

In consequence of the previously discussed morphological limitations of our experimental hexapod, RHex, leg motion during swing takes the form of recirculation rather than protraction. Consequently, we use a "mirror law" [6] to determine the target angle for the swing leg, which is then tracked by a local PD style feedback controller. The resulting feedback law is purely a function of the system state and ensures accurate and timely placement of the touchdown leg.

3 Hexapedal Running

3.1 The Spatial Hexapod Model

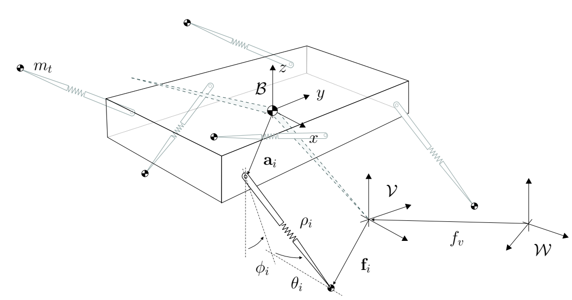

Figure 2: The compliant hexapod model.

Figure 2 illustrates our spatial compliant hexapod model. Three reference frames are defined: $\mathcal{W}$ as the fixed inertial world frame, $\mathcal{V}$ as the virtual toe frame, located at the foot of a "virtual leg" and finally $\mathcal{B}$ as the body frame, affixed to the center of mass of the system. $\mathcal{V}$ and $\mathcal{W}$ have the same orientation except a yaw rotation around the z axis. The orientation of the body is determined by the yaw γ, pitch α and the roll β angular degrees of freedom.

The model consists of a rigid body and six compliant legs with fixed attachment points. Each leg has small toe mass to capture its flight dynamics as well as radial and sideways torsional springs and viscous dampers. For legs in stance, the toe positions $\mathbf{f}_i$ are fixed on the ground. In contrast, legs that are in flight do not exert forces on the body. Instead, the motion of the leg is governed by the toe mass under the influence of the leg forces. Moreover, the position and velocity of toe masses in flight become independent coordinates.

The morphology of this model accurately captures RHex's design. There are, however, two major differences in its dynamical properties. Firstly, the assumption that the toes remain stationary during stance is rather unrealistic. In fact, particularly at high speeds, leg slippage is one of the major limiting factors in RHex's performance. Secondly, simple linear leg compliance and damping models that we adopt are not experimentally verified and are likely to be inaccurate.

3.2 The Structure of the Controller

The hexapedal running controller closely parallels the bipedal controller of Section 2.2. We associate each tripod with one of the biped legs and use the same FSM to impose an alternating tripod gait. Furthermore, the same gait level deadbeat controller is used to determine the desired touchdown commands at each step. There are, however, a few significant differences in the remaining components.

First of all, active radial actuation of the legs is not possible in RHex. As a consequence, it is not as simple to achieve the desired touchdown precompression. Toward this end, we introduce the idea of a "virtual toe" in Section 3.3, whose explicit placement in combination with appropriate modifications on the recirculation control yields the desired touchdown commands.

The remaining differences relate to the control of the stance (active) tripod. Section 3.4 briefly presents how we achieve the embedding of the BSLIP template through active control of the stance tripod.

3.3 Virtual Toe Placement and Coordinates

At each step, our gait level BSLIP controller commands leg touchdown states to regulate the running speed and height. These commands must be realized by the underlying mechanism to yield convergence to the desired gait. Unfortunately, our limited actuation affordance over the hexapod does not admit precompression of its legs. It is hence unclear how to realize the touchdown commands of the gait controller.

Our solution is to introduce the idea of a virtual toe, distinct from the physical toes of the hexapod. This also defines a virtual leg between the toe and the body center of mass, establishing a natural connection to individual legs of the biped.

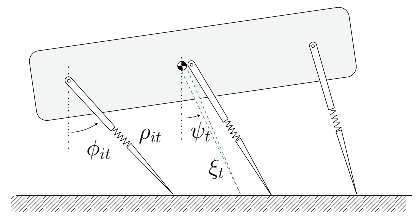

Figure 3: Kinematics of touchdown.

We use recirculation of the swing legs in conjunction with proper placement of the virtual toe to achieve the desired BSLIP touchdown states. Given the commanded leg angle ψ_t and precompression ξ_t as well as the current body orientation, it is possible to solve the kinematics to compute target angles for the swing legs of the hexapod (see Figure 3). Our recirculation controller for the hexapod takes the form of a mirror law, designed to achieve these target angles precisely at the moment of touchdown, while both avoiding premature transition into stance and satisfying the commands of the gait level controller[1].

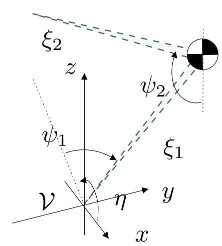

Figure 4: Virtual foot coordinates.

The placement of the virtual foot also determines the new origin for the virtual toe frame $\mathcal{V}$. In order to facilitate the embedding of the BSLIP template, we define a new spherical coordinate system within $\mathcal{V}$: virtual toe coordinates (see Figure 4).

3.4 Active Embedding of the Template

The goal of the embedding controller is to choose appropriate hip torque controls such that the dynamics of the hexapod center of mass mimic the passive stance dynamics of BSLIP as accurately as possible. The result is an effective reduction of the hexapod dynamics to the much simpler template dynamics, yielding the

[1]See [17] for details of the derivations

ability to regulate speed and height of locomotion using the gait level BSLIP controllers.

We start by deriving the dynamics in virtual toe coordinates. Defining $\mathbf{c} := [\ \xi, \psi, \eta, \gamma, \alpha, \beta\ ]$, we have

$$\mathbf{M}(\mathbf{c})\ddot{\mathbf{c}} = f(\mathbf{c}, \dot{\mathbf{c}}) + \mathbf{K}\ , \tag{1}$$

where $f(\mathbf{c}, \dot{\mathbf{c}})$ represents the unforced dynamics in polar coordinates and the forcing vector $\mathbf{K}$ is defined as

$$\mathbf{K} := (D_{\mathbf{c}}\boldsymbol{\phi})\boldsymbol{\tau} + (D_{\mathbf{c}}\boldsymbol{\rho})\mathbf{F}_r + (D_{\mathbf{c}}\boldsymbol{\theta})\boldsymbol{\tau}_\theta\ , \tag{2}$$

with $\boldsymbol{\tau}$, $\mathbf{F}_r$ and $\boldsymbol{\tau}_\theta$ representing the hip actuation, radial spring force and sideways torque vectors.

For exact embedding of BSLIP within the hexapod, we require a forcing vector of the form

$$\mathbf{K} = \left[U^*(\xi),\ 0,\ 0,\ M^*_\gamma,\ M^*_\alpha,\ M^*_\beta\right]\ ,$$

where $U^*(\xi)$ denotes the potential law for the radial BSLIP spring. M^*_γ, M^*_α and M^*_β are desired effective torques on the Euler angle coordinates of the body orientation and are chosen through simple PD laws to stabilize the body to its neutral orientation.

The most obvious solution to (2) would be through inversion of $D_{\mathbf{c}}\boldsymbol{\phi}$. However, this turns out to be infeasible for a variety of reasons. First of all, the alternating tripod gait imposed by our controller admits at most three legs in contact with the ground at any time, yielding an underactuated system. Furthermore, our recirculation based strategy usually forces the system to go through configurations where all legs of the stance tripod are approximately parallel, decreasing control affordance. Finally, the hexapod almost always goes through midstance with vertical leg configurations and neutral body orientation, a singularity which is even more restrictive.

In order to address these problems, we propose a partial inversion of the dynamics. To this end, the structure of $D_{\mathbf{c}}\boldsymbol{\phi}$ suggests certain reductions (see [17] for a detailed discussion). In particular, we disregard the radial extension ξ as well as the sideways translation η from the inversion as the hip actuation offers very little if no affordance over these coordinates.

For practical applicability of our controller design, we impose a magnitude limit on the hip torques to match RHex's commercial actuator specifications. We also attempt to keep the stance legs on the ground as long as possible to avoid losing control affordance, imposing unilateral constraints on the allowable torque commands for each leg. The combination of these constraints yields the allowable torque space, defined as

$$\mathcal{T} := \{\ \boldsymbol{\tau}\ \mid\ \tau_{i,min} \leq \tau_i \leq \tau_{i,max}\ \}\ . \tag{3}$$

We now define two subsets of the virtual toe coordinates, $\mathbf{c}_1 := [\ \psi,\ \alpha,\ \beta\ ]^T$ and $\mathbf{c}_2 := [\ \psi,\ \gamma\ ]^T$. In inverting the associated submatrices of the overall jacobian, we prioritize the saggital plane angle ψ as the quality of the embedding is directly affected by the accuracy with which this component is satisfied. We hence require feasible torque solutions to lie in the set

$$\mathcal{T}_\psi := \{\ \boldsymbol{\tau}\ \mid\ (D_\psi\boldsymbol{\phi})\boldsymbol{\tau} + B_\psi = 0\ \}\ . \tag{4}$$

where B_ψ represents additional terms in (1).

Prior to computing the final solution, we first examine the determinant of $D_{\mathbf{c}_1}\boldsymbol{\phi}$. We use the submatrix corresponding to $\mathbf{c}_1$ whenever $\det(D_{\mathbf{c}_1}\boldsymbol{\phi}) > d_{min}$, and to $\mathbf{c}_2$ otherwise. In both cases, the final torque solution is computed by projecting the unconstrained solution from the jacobian inversion onto the allowable torque space of (3) along the feasible subspace defined in (4)[2].

4 Simulation Studies

In this section, we use numerical simulations to show that our template based controller achieves asymptotically stable locomotion for a wide range of forward speeds. We also characterize in simulation, stability properties of the associated limit cycles.

The results of the following sections were obtained using kinematic and dynamic parameters that match RHex's morphology as closely as possible [17]. Despite differences in the surface contact model as well as the lack of experimental validation of our leg compliance and damping models, we believe that the simulation results we present will be qualitatively applicable to physical implementations.

4.1 The Nature of Stable Orbits

The action of the template based controller on the spatial hexapod model results in a completely autonomous dynamical system devoid of all time dependency. We have been able to identify through simulation, asymptotically stable limit cycles of this system which seem to be unique for each different gait level goal setting. Furthermore, these limit cycles all seem to have the same structure and characteristic features.

First of all, despite the mirrored morphology of the left and right tripods, the projection of the limit cycle onto the saggital plane coordinates exhibits period one behavior from one step to the next. Figure 5 illustrates this aspect of an example run.

On the other hand, projection onto the roll and yaw degrees of freedom reveals period two behavior as a consequence of the alternation between the left and right tripods. Fortunately, this does not seem to affect the task level stability in the saggital plane coordinates, which were always observed to be period

[2] See [17] for details on how this projection is performed.

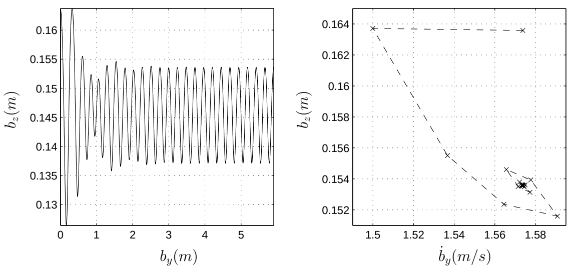

Figure 5: The attracting limit cycle for an example run with $\dot{b}_y^* = 1.6m/s$. *Left*: the saggital plane robot position; *Right*: the progression of the gait level apex state towards the fixed point in the $\dot{b}_y - b_z$ plane.

one. All the limit cycles we have obtained using the template based controller exhibit these properties.

4.2 Stability and Basins of Attraction

In this section, we present simulation evidence to show that the stable limit cycle described above can be effectively adjusted by changing gait level commands to the SLIP template, resulting in an effective control of forward velocity. Specifically, we summarize the results of careful numerical study indicating that the basins of attraction associated with these cycles are sufficiently large to admit smooth control of forward velocity of locomotion.

By considering certain symmetries in the system, it is possible to reduce the 12 dimensional space of initial conditions. In particular, we do not need to consider the horizontal translation and yaw initial conditions of the robot. Furthermore, since gait characteristics are well captured by the discrete return map, we drop another dimension (choosing to work with "apex" coordinates that specifically eliminate the vertical velocity component). As a consequence, the dimension of the space of initial conditions is reduced to eight. However, even with this reduced space, it is very costly to characterize carefully the basins of attraction due to the computational cost of the required simulations. For purposes of presentation we project onto two different pairwise combinations of these eight dimensions, an approximation to the basin around four different speed settings, illustrated in Figures 6 and 7.

In particular, Figure 6 concerns the saggital plane velocity and height, which also correspond to the task level coordinates of the BSLIP template. The surprisingly large basins of attraction associated with four different speed settings suggest that smooth control of forward velocity is possible.

Similarly, Figure 6 illustrates the projection of the basin of attraction onto two most critical orientational degrees of freedom of the rigid body. Even though roll instability is the predominant source of failure for our controller, the basins of attraction are reasonably large in both the pitch and roll directions. This relatively strong stability suggests that practical implementations on RHex may be feasible.

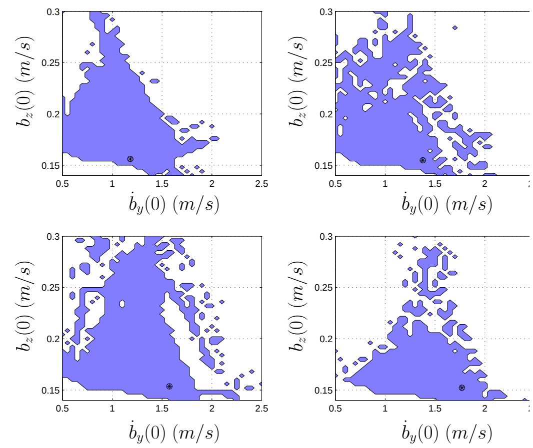

Figure 6: Basin of attraction for 4 different speed goals $\dot{b}_y^* \in \{1.2, 1.4, 1.6, 1.8\}$m/s in BSLIP apex coordinates. For each goal setting, the filled circle indicates the stable limit cycle.

5 Conclusion

In this paper, we develop a novel model based locomotion controller that is capable of achieving asymptotically stable hexapedal running for a large range of speeds. We demonstrate the efficacy of this controller by its application to a hybrid Lagrangian model of the hexapedal robot, RHex [18]. A bipedal extension of the well-studied Spring-Loaded Inverted Pendulum model is used as the dynamical motion template and command interface for hexapedal locomotion. This is complemented by an inverse dynamics style controller designed to embed the template dynamics within the hexapod model, effectively reducing the model dynamics to those of the template. Simulation studies yield convincing evidence that the combination of a gait level controller acting on the template, with our model based embedding strategy is sufficient to achieve asymptotically stable hexapedal running.

The natural next step is implementation on RHex. However, there are a number of challenges in realizing such an implementation. In particular, more accurate extensions to the current simplistic ground contact and passive leg models are needed to ensure practical applicability of the controller . Some recent results promise to address some of these issues [13].

Further difficulties arise from the high bandwidth

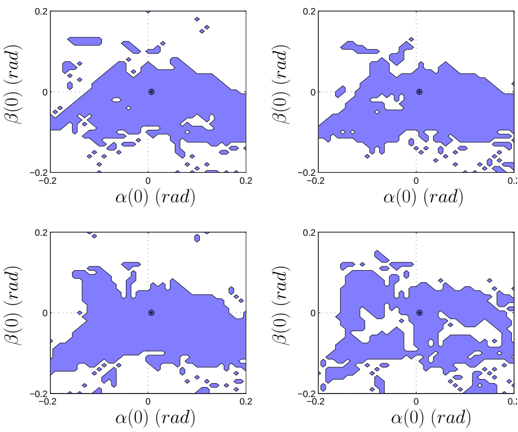

Figure 7: Basin of attraction for 4 different speed goals $\dot{b}_y^* \in \{1.2, 1.4, 1.6, 1.8\}$m/s in pitch and roll coordinates. For each goal setting, the filled circle indicates the stable fixed point.

state feedback required by our controller. Clearly, any implementation of such an algorithm requires accurate and reliable sensing that will be feasible only after significant effort, now in progress, devoted to the design and implementation of careful state estimators over RHex's expanding sensor suite. A careful characterization of our controller's performance under noise as well as the implementation of correspondingly accurate sensor hardware and software for RHex needs to be completed prior to experimental implementation.

In summary, there is still a long research path to our goal of building a fully autonomous legged platform capable of surviving a large range of outdoor environments for extended periods of time. However, we believe that our work represents an important step in this direction and begins to develop some of the tools and concepts that are necessary to achieve this goal.

Acknowledgments

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